Available online at www.sciencedirect.com
ScienceDirect
Procedia Computer Science 47 (2015) 337 – 341
Procedia Computer Science

The Minimum Eccentric Dominating Graph

M. Bhanumathi[a], J. John Flavia[b]

a Government Arts College for Women,Pudukkottai-622001, TN, India

b Government Arts College for Women,Pudukkottai-622001, TN, India

Abstract

The minimum eccentric dominating graph $MmD_E(G)$ of a graph G is defined to be the intersection graph on the minimum eccentric dominating sets of G. In this paper minimum eccentric dominating graph of some families of graphs are studied.

Peer-review under responsibility of organizing committee of the Graph Algorithms, High Performance Implementations and Applications (ICGHIA2014)

Keywords: Dominating graph; Eccentric dominating set; Minimum eccentric dominating graph;

1. Introduction

Let G be a finite, simple, undirected graph on n vertices with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary [4] Buckley and Harary [2].

Definition 1.1 Let G be a connected graph and u be a vertex of G. The **eccentricity** e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u,v) : u \in V\}$. The **radius** r(G) is the minimum eccentricity of the vertices, whereas the **diameter** diam(G) = d(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. v is a central vertex if e(v) = r(G).

E-mail address:bhanu_ksp@yahoo.com[a], kavisdev@gmail.com[b]

Peer-review under responsibility of organizing committee of the Graph Algorithms, High Performance Implementations and Applications (ICGHIA2014)
doi:10.1016/j.procs.2015.03.214

For a vertex v, each vertex at a distance e(v) from v is an **eccentric vertex of v** . **Eccentric set of a vertex** v is defined as $E(v) = \{u \in V(G) / d(u,v) = e(v)\}$.

Definition 1.2 The **open neighborhood** N(u) of a vertex v is the set of all vertices adjacent to v in G. $N[v] = N(v)\cup\{v\}$ is called the **closed neighborhood** of v.

Definition 1.3 A **bigraph or bipartite graph** G is a graph whose point set V can be partitioned into two subsets V_1 and V_2 such that every line of G joins V_1 with V_2. If further G contains every line joining the points of V_1 to the points of V_2 then G is called a **complete bigraph**. If V_1 contains m points and V_2 contains n points then the complete bigraph G is denoted by $K_{m,n}$.

Definition 1.4 A **star** is a complete bi graph $K_{1,n}$.

Definition 1.5[3] A set $D \subseteq V(G)$ is said to be a **dominating set** in G, if every vertex in $V-D$ is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the **domination number** and is denoted by $\gamma(G)$.

Definition 1.6[5] A set $D \subseteq V(G)$ is an **eccentric dominating set** if D is a dominating set of G and for every $v \in V-D$, there exists at least one eccentric point of v in D.

If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set.

An eccentric dominating set D is a **minimal eccentric dominating set** if no proper subset $D'' \subseteq D$ is an eccentric dominating set. The minimum cardinality of an eccentric dominating set is called the **eccentric domination number** and is denoted by $\gamma_{ed}(G)$. An eccentric dominating set with cardinality $\gamma_{ed}(G)$ is known as **minimum eccentric dominating set**.

Definition 1.7[6] Let S be a finite set and let $F = \{S_1, S_2,\ldots, S_n\}$ be a partition of S. Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices S_i and S_j are adjacent if and only if $S_i \cap S_j \neq \phi$.

Definition 1.8 We define $\gamma_{ED}(G)$ as the total number of minimum eccentric dominating sets in a graph G.

Theorem: 1.1[6] $\gamma_{ed}(K_n) = 1$.

Theorem: 1.2[6] $\gamma_{ed}(K_{m,n}) = 2$.

Theorem: 1.3[6] $\gamma_{ed}(K_{1,n}) = 2, n \geq 2$.

Theorem: 1.4[6] $\gamma_{ed}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, & if\ n = 3k+1 \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ n = 3k\ or\ 3k+2. \end{cases}$

Theorem: 1.5[1] (i) $\gamma_{ED}(P_{3k}) = k.$

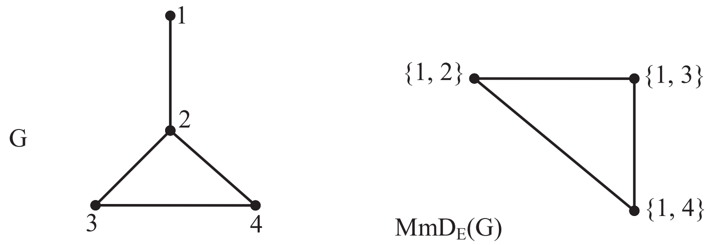
(ii) $\gamma_{ED}(P_{3k+1}) = 1.$

(iii) $\gamma_{ED}(P_{3k+2}) = \frac{k^2+3k+2}{2}.$

2. The Minimum eccentric dominating graph

Definition 2.1 The minimum eccentric dominating graph $MmD_E(G)$ of a graph is the intersection graph defined on the family of all minimum eccentric dominating sets of vertices in G.

Example: 2.1



{1, 2}, {1, 3}, {1, 4} are the minimum eccentric dominating sets of G.

Theorem: 2.1

If G is a complete graph with $n \geq 2$ vertices, then $MmD_E(G)$ is totally disconnected.

Proof:

Let G be a complete graph with $n \geq 2$ vertices. Then each vertex in G is a minimum eccentric dominating set. Thus every pair of vertices in $MmD_E(G)$ is not adjacent. Hence $MmD_E(G)$ is totally disconnected.

Theorem: 2.2

$\gamma_{ed}(MmD_E(K_n)) = n.$

Proof:

$MmD_E(K_n)$ is totally disconnected. Hence $\gamma_{ed}(MmD_E(K_n)) = n.$

Theorem: 2.3

If G is a star $K_{1,n}$, $n \geq 2$, then $MmD_E(G) = K_n$.

Proof:

Suppose $G = K_{1,n}$, $n \geq 2$ such that v is a central vertex with deg $v = n$ and deg $v_i = 1$, $1 \leq i \leq n$. Then the pairs (v, v_i) $1 \leq i \leq n$ of vertices in G form minimum eccentric dominating sets. Totally there are n such minimum eccentric dominating sets. In $MmD_E(G)$ each vertex is adjacent to other vertices. Hence $MmD_E(G)$ is complete and $MmD_E(K_{1,n}) = K_n$.

Theorem: 2.4

$\gamma_{ed}(MmD_E(K_{1,n})) = 1$.

Proof:

$MmD_E(K_{1,n})$ is a complete graph. Hence $\gamma_{ed}(MmD_E(K_{1,n})) = 1$.

Theorem: 2.5

$MmD_E((K_{2n}-1 \text{ factor of } K_{2n})) = K_{2^n} - 1$ factor of K_{2^n} .

Proof:

Let G be obtained from the complete graph K_{2n} by deleting edges of a linear factor. Let u_i and u'_i for i = 1, 2, 3,…, n be a pair of non-adjacent vertices in G. Then u_i and u'_i are eccentric to each other. $\{u_1, u_2, u_3,\ldots, u_n\}$ and $\{u'_1, u'_2, u'_3,\ldots, u'_n\}$ are the disjoint minimum eccentric dominating sets. Similarly, we can form 2^n minimum eccentric dominating sets. Therefore, In $MmD_E(G)$, there are 2^n vertices and two of them are non adjacent if and only if they are disjoint. Thus we see that $MmD_E(G) = K_{2^n} - 1$ factor.

Hence $MmD_E((K_{2n}-1 \text{ factor of } K_{2n})) = K_{2^n} - 1$factor of K_{2^n} .

Theorem: 2.6

$MmD_E(P_{3k}) = K_k$, $MmD_E(P_{3k+1}) = K_1$, $MmD_E(P_{3k+2}) = K_l$, where $l = \dfrac{k^2+3k+2}{2}$

Proof:

Let G = P_n. An eccentric dominating set of P_n must contain the two end vertices.

Case (i) n = 3k

Number of minimum eccentric dominating sets in P_{3k} is k. If k = 1, any two minimum eccentric dominating sets contain a common vertex. In $MmD_E(G)$, $D_i \cap D_j \neq \phi$. Therefore $MmD_E(G)$ is complete. If $k \geq 2$, every minimum eccentric dominating set contains the end vertices of P_n. Hence, in $MmD_E(G)$, each vertex is adjacent to other vertices. Therefore $MmD_E(G)$ is complete and $MmD_E(P_{3k}) = K_k$.

Case (ii) n = 3k+1

Number of minimum eccentric dominating set in P_{3k+1} is one. $MmD_E(G)$ is an isolated vertex. Hence $MmD_E(P_{3k+1}) = K_1$.

Case (iii) n = 3k+2

Number of minimum eccentric dominating sets in P_{3k+2} is $\dfrac{k^2+3k+2}{2}$ (say l). Every minimum eccentric dominating set contains the end vertices of P_n. Hence, In $MmD_E(G)$, each vertex is adjacent to other vertices. Therefore, $MmD_E(G)$ is complete and $MmD_E(P_{3k+2}) = K_l$, where $l = \dfrac{k^2+3k+2}{2}$

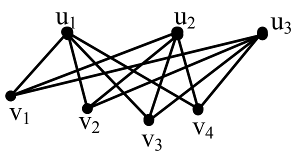
Theorem 2.7

$MmD_E(K_{m,\,n})$ is a m+n-2 regular graph on mn vertices. Also, $MmD_E(K_{m,\,n}) = K_m \times K_n$.

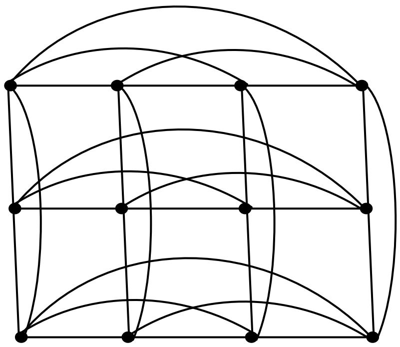
Proof:

Let $G = K_{m,\,n}$, $V(G) = V_1 \cup V_2$. $V_1 = \{v_1, v_2, v_3... v_m\}$ and $V_2 = \{u_1, u_2, u_3... u_n\}$. $\{v_i, u_j\}$ for i = 1, 2, ..., m and j = 1, 2, ..., n form the minimum eccentric dominating set. Therefore, we get mn minimum eccentric dominating sets. In $MmD_E(G)$, each vertex is non-adjacent to (n-1)(m-1) vertices and is adjacent to(m-1) + (n-1) vertices. Hence, degree of each vertex is (m-1) + (n-1). Hence $MmD_E(K_{m,\,n})$ is a m+n-2 regular graph and this graph is isomorphic to $K_m \times K_n$.

Example:2.2



G = $K_{3,4}$



$MmD_E(K_{3,4}) = K_3 \times K_4$

References:

[1]Bhanumathi M and John Flavia J, Minimum Eccentric dominating sets in paths and cycles, communicated to International Journal of Engineering Science, advanced Computing and Bio-Technology.

[2] Buckley. F, Harary. F, Distance in graphs, Addison–Wesley, Publishing Company (1990).

[3] Cockayne, E.J., Hedetniemi, S.T., Towards a theory of domination in graphs. Networks, 7:247-261,1977.

[4] Harary, F., Graph theory, Addition - Wesley Publishing Company Reading, Mass (1972).

[5] Janakiraman T.N., Bhanumathi M and Muthammai S, Eccentric domination in graphs, International Journal of Engineering Science, advanced Computing and Bio-Technology, Volume 1, No.2, pp 1-16, 2010.

[6] Kulli V.R, Theory of domination in graphs, Vishwa International publications (2010).